CC III

Q1 :

2 pts $$F(x,T) = G_+(cT + X) + G_-(cT - X)$$

avec $G_+, G_-$ des fonctions arbitraires.

Q2 :

on pose $F(x,T) = e^{i(\omega T - kx)}$

3 pts

$$\Rightarrow \frac{-\omega^2}{c^2} + k^2 + m^2c^2 = 0$$

$$\Rightarrow \boxed{\omega^2 = c^2(k^2 + m^2c^2)}$$ Rel. de disp.

Vitesse de phase : $V_\phi = \omega/k = c\sqrt{1 + \frac{m^2c^2}{k^2}}$

Vitesse de groupe : $V_g = \frac{d\omega}{dk} = \frac{c^2 k}{\sqrt{k^2 + m^2c^2}} = \frac{c^2}{V_\phi}$

## Q III :

On pose $F(x,y,T) = A(x)\,B(y)\,C(T)$ eT

3 pts

l'éq. devienT

$$\frac{1}{\alpha^2} ABC'' - A''BC - AB''C = 0$$

$$\Rightarrow \underbrace{\frac{1}{\alpha^2}\frac{C''}{C}}_{\text{Fonction de } T} - \underbrace{\frac{A''}{A}}_{\text{Fonction de } x} - \underbrace{\frac{B''}{B}}_{\text{Fonction de } y} = 0$$

$\Rightarrow$ Chaque Terme doiT êTre constanT !

$$\Rightarrow \left.\begin{aligned} C'' &= -\omega^2 C \\ A'' &= -k_x^2 A \\ B'' &= -k_y^2 B \end{aligned}\right\} \quad \boxed{\omega^2 = \alpha^2 \left(k_x^2 + k_y^2\right)}$$

Q Longue :

a) On pose $F(x,T) = e^{i(\omega T - kx)}$.

2 pts

$$\Rightarrow \frac{\mu}{T_0}\omega^2 - k^2 = 0 \Rightarrow \omega^2 = \frac{T_0 k^2}{\mu}$$

$$\Rightarrow \boxed{V_\phi = \frac{\omega}{k} = \sqrt{\frac{T_0}{\mu}}}$$

b)

Ansatz: $F(x,T) = e^{i\omega T}\left(A\cos(kx) + B\sin(kx)\right)$

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on a bien $\frac{\partial^2}{\partial T^2}F = -\omega^2 F$ , $\frac{\partial^2 F}{\partial x^2} = -k^2 F$

$\Rightarrow$ si $\omega^2 = \frac{T_0 k^2}{\mu}$, c'est bien une sol !

On doit avoir $F(0,T) = F(L,T) = 0$ $\Rightarrow$

$$\Rightarrow A = 0$$

$$B\sin kL = 0 \Rightarrow k = \frac{n\pi}{L}, \quad n = 1, \dots, \infty$$

$$\Rightarrow F(x,T) = \sum_{n=1}^{\infty}\left(a_n \cos\omega_0 nT + b_n \sin\omega_0 nT\right)\sin\left(\frac{n\pi x}{L}\right)$$

$\omega_0 n = \sqrt{\frac{T_0}{\mu}}\frac{n\pi}{L}$ $\Rightarrow$ $\boxed{\omega_0 = \sqrt{\frac{T_0}{\mu}}\frac{\pi}{L}}$

c) On a

~~4~~ 3 $$F(x,0) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right)$$

$$\Rightarrow \boxed{a_n = \frac{2}{L}\int_0^L \sin\left(\frac{n\pi x}{L}\right) F(x,0)\,dx}$$

$$\left.\frac{\partial F(x,T)}{\partial T}\right|_{T=0} = \sum_{n=0}^{\infty} b_n \omega_0 n \sin\left(\frac{n\pi x}{L}\right)$$

$$\Rightarrow \boxed{b_n = \frac{2}{nL\omega_0}\int_0^L \sin\left(\frac{n\pi x}{L}\right) \left.\frac{\partial F(x,T)}{\partial T}\right|_{T=0} dx}$$

d) $$\left.\frac{\partial F}{\partial T}\right|_{T=0} = 0 \Rightarrow \boxed{b_n = 0}$$

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3 $$a_n = \frac{2}{L}\int_0^{L/2} x \sin\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{2}{L}\left[ -\frac{L}{n\pi}\, x\cos\left(\frac{n\pi x}{L}\right)\Big|_0^{L/2} + \frac{L}{n\pi}\int_0^{L/2} \cos\left(\frac{n\pi x}{L}\right) dx \right]$$

$$\Rightarrow \quad a_n = -\frac{L}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{L^2}{n^2\pi^2}\sin\left(\frac{n\pi x}{L}\right)\Bigg|_0^{L/2}$$

$$\Rightarrow \quad \boxed{a_n = -\frac{L}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{L^2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)}$$