Corrigé CC2 - ondes

[1] 1pt ① $f_{n+1\to n} = -K_0(x_n - x_{n+1})$

1pt ② $f_{n-1\to n} = -K_0(x_n - x_{n-1})$

2pts ③ $m\ddot{x}_n = -K_0(x_n - x_{n+1}) - K_0(x_n - x_{n-1})$

$$\ddot{x}_n + w_0^2(2x_n - x_{n-1} - x_{n+1}) = 0$$

[2]

2pts ① $\begin{cases} \ddot{x}_1 + w_0^2(2x_1 - x_2) = 0 & x_0 = 0 \\ \ddot{x}_2 + w_0^2(2x_2 - x_1) = 0 & x_3 = 0 \end{cases}$

2pts ② $\dfrac{d^2\vec{X}}{dt^2} = A\vec{X}$ $\quad A = \begin{pmatrix} -2w_0^2 & w_0^2 \\ w_0^2 & -2w_0^2 \end{pmatrix} = w_0^2\begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$

2pts ③ $A\vec{X} = \lambda\vec{X}$ $\quad \det(A - \lambda \, Id_{2\times 2}) = 0$

$\lambda_1 = -w_0^2$ et $\lambda_2 = -3w_0^2$

2pts ④ $A\vec{V_1} = \lambda_1\vec{V_1} \ldots \vec{V_1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

[Remarque : $\vec{V_1}'$ (non normalisée) $= \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ est aussi accepté]

Même chose $\vec{V_2} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\quad \left(\vec{V_2}' = \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right)$

5a) |4 pts|

2/3

$$\begin{cases} x_1(0) = \frac{1}{\sqrt{2}}(\alpha + \beta + \gamma + \delta) = a_0 & (1) \\ x_2(0) = \frac{1}{\sqrt{2}}(\alpha + \beta - \gamma - \delta) = -a_0 & (2) \end{cases} \rightarrow \alpha = -\beta$$

1pt

$$\dot{\vec{X}}(0) = i\alpha w_0 \vec{V_1} - i\beta w_0 \vec{V_1} + i\sqrt{3} w_0 \gamma \vec{V_2} - i\sqrt{3} w_0 \delta \vec{V_2} = \vec{0}$$

$$\alpha \vec{V_1} - \beta \vec{V_1} + \sqrt{3}\gamma \vec{V_2} - \sqrt{3}\delta \vec{V_2} = \vec{0}$$

1pt

$$\begin{cases} \alpha - \beta + \sqrt{3}\gamma - \sqrt{3}\delta = 0 & (3) \\ \alpha - \beta - \sqrt{3}\gamma + \sqrt{3}\delta = 0 & (4) \end{cases} \rightarrow \alpha = \beta$$

$\alpha = -\beta$ et $\alpha = \beta \rightarrow \alpha = \beta = 0$

$(1) - (2) \rightarrow \cancel{2}(\gamma + \delta) = \cancel{2} a_0 \sqrt{2}$

$(3) - (4) \rightarrow \cancel{2\sqrt{3}}\gamma - \cancel{2\sqrt{3}}\delta = 0 \qquad \delta = \gamma$

$(1) \rightarrow \frac{2\delta}{\sqrt{2}} = a_0 \rightarrow \boxed{\delta = \frac{a_0}{\sqrt{2}} = \gamma}$

$$\vec{X} = \frac{a_0}{\sqrt{2}} e^{i w_2 t} \vec{V_2} + \frac{a_0}{\sqrt{2}} e^{-i w_2 t} \vec{V_2}$$

$$\vec{X} = \frac{a_0}{2}\left(e^{i w_2 t} + e^{-i w_2 t}\right)\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

2pts

$$\begin{cases} x_1(t) = a_0 \cos(w_2 t) \\ x_1(t) = -a_0 \cos(w_2 t) \end{cases} \qquad w_2 = \sqrt{3} w_0$$

4pts

5b

$$\begin{cases} x_1(0) = \frac{1}{\sqrt{2}}(\alpha+\beta+\gamma+\delta) = a_0 & (1) \\ x_2(0) = \frac{1}{\sqrt{2}}(\alpha+\beta-\gamma-\delta) = a_0 & (2) \end{cases}$$

1pt

$$\begin{cases} \alpha-\beta+\sqrt{3}\gamma-\sqrt{3}\delta = 0 & (3) \\ \alpha-\beta-\sqrt{3}\gamma+\sqrt{3}\delta = 0 & (4) \end{cases} \rightarrow \boxed{\alpha=\beta}$$

1pt

$$(1)-(2) \rightarrow \gamma+\delta = 0 \rightarrow \boxed{\gamma=-\delta}$$

$$\frac{2\alpha}{\sqrt{2}} = a_0 \qquad \alpha=\beta=\frac{a_0}{\sqrt{2}}$$

$$(3)-(4) \rightarrow 2\sqrt{3}\gamma-2\sqrt{3}\delta = 0$$

$$\boxed{\gamma-\delta=0} \rightarrow \gamma=\delta$$

d'où $\gamma=\delta=0$

$$\vec{X}(t) = \left(\frac{a_0}{\sqrt{2}}\cdot e^{i\omega_1 t} + \frac{a_0}{\sqrt{2}}\cdot e^{-i\omega_1 t}\right)\vec{V_1}$$

$$\vec{X}(t) = a_0 \cos(\omega_1 t)\begin{pmatrix}1\\1\end{pmatrix}$$

$$\begin{cases} x_1(t) = a_0\cos(\omega_1 t) \\ x_2(t) = a_0\cos(\omega_1 t) \end{cases}$$

2pts