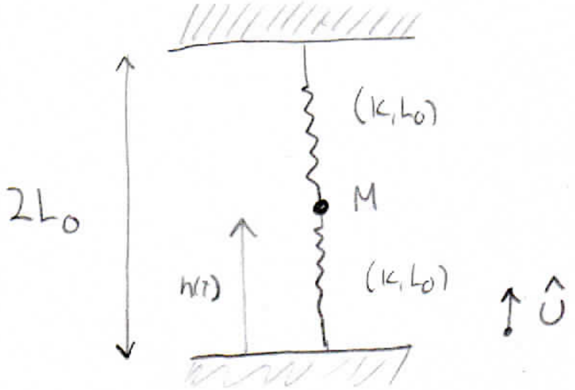

Partie I

2 a) Bilan des forces

Ressort bas : $\vec{F}_1 = -k\,(h(t) - L_0)\,\hat{u}$ $\qquad \vec{F}_g = -Mg\,\hat{u}$

Ressort haut : $\vec{F}_2 = -k\,(L_0 - (2L_0 - h))\,\hat{u}$

$= -k\,(h(t) - L_0)\,\hat{u}$

$$\boxed{\vec{F}_{TOT} = -2k\,(h(t) - L_0)\,\hat{u} - Mg\,\hat{u}}$$

3 b) Eq. du mouvement

$$\vec{F}_{TOT} = M\vec{a} = M\ddot{h}\,\hat{u}$$

$$\Rightarrow \quad M\ddot{h} + 2kh = 2kL_0 - Mg$$

$$\Rightarrow \quad \ddot{h} + \underbrace{\frac{2k}{M}}_{\omega_0^2} h = \underbrace{\frac{2kL_0}{M} - g}_{h_0}$$

$\omega_0 = \sqrt{2k/M}$

$h_0 = \frac{2kL_0}{M} - g$

2 c) Sol. générale :

On cherche une sol particulière de l'équation et y ajoute la sol. gén. du prob. homogène.

Prob. homogène : $\ddot{h} + \omega_0^2 h = 0$

$$\Rightarrow \quad h = A \cos \omega_0 T + B \sin \omega_0 T$$

Sol. particulière ; Ansatz : $h = Z \equiv$ constante

$$\Rightarrow \quad \ddot{h} + \omega_0^2 h = \omega_0^2 Z = h_0 \iff Z = h_0/\omega_0^2$$

Sol. gén. $\boxed{h(T) = h_0/\omega_0^2 + A \cos \omega_0 T + B \sin \omega_0 T}$

A, B des constantes arbitraires

d) Prob. de Cauchy, $h(0) = L_0$, $\dot{h}(0) = 0$
3

$$\Rightarrow \quad h(0) = h_0/\omega_0^2 + A \cos(0) + B \sin(0)$$

$$\Rightarrow \quad \boxed{A = L_0 - \frac{h_0}{\omega_0^2}} = L_0 - \frac{(\omega_0^2 L_0 - g)}{\omega_0^2} = g/\omega_0^2$$

$$\dot{h}(0) = B\omega_0 = 0 \quad \Rightarrow \quad B = 0$$

$$\Rightarrow \boxed{h(T) = \frac{h_0}{\omega_0^2} + \left(L_0 - \frac{h_0}{\omega_0^2}\right)\cos\omega_0 T}$$

Partie II

2 e) On ajoute $\vec{F} = -\gamma\vec{V} = -\gamma\dot{h}\hat{u}$

$$\Rightarrow \quad M\ddot{h} = 2K(L_0 - h) - Mg - \gamma\dot{h}$$

$$\Rightarrow \boxed{\ddot{h} + \underbrace{\frac{\gamma}{M}}_{\Gamma}\dot{h} + \underbrace{\frac{2K}{M}}_{\omega_0^2}h = \underbrace{\frac{2K}{M}L_0 - g}_{h_0}}$$

$$\boxed{\Gamma = \gamma/M}$$

3 f) $h(T) = A + Be^{-i\alpha T} \Rightarrow \dot{h} = -i\alpha Be^{-i\alpha T}$, $\ddot{h} = -\alpha^2 Be^{-i\alpha T}$

$$\Rightarrow (-\alpha^2 - i\alpha\Gamma + \omega_0^2)Be^{-i\alpha T} + A\omega_0^2 = h_0$$

$\Rightarrow \boxed{A = h_0/\omega_0^2}$

$\alpha^2 + i\alpha\Gamma - \omega_0^2 = 0 \Rightarrow \boxed{\alpha = \frac{-i\Gamma}{2} \pm \sqrt{\omega_0^2 - \Gamma^2/4}}$

2 g) critique $\Leftrightarrow \boxed{\omega_0^2 = \Gamma^2/4}$

3 h) $\omega_0^2 = (\Gamma/2)^2 + 4 \Rightarrow \alpha = -i\Gamma/2 \pm \sqrt{4}$

$= -i\Gamma/2 \pm 2$

$\Rightarrow \boxed{h(T) = \frac{h_0}{\omega_0^2} + e^{-\Gamma T/2}(A \cos 2T + B \sin 2T)}$