DS I

Questions courtes /8

1 QI: $\omega = 2\pi f$, $f = 1/\tau$ $\Rightarrow$ $\boxed{\omega = \frac{2\pi}{\tau}}$

2 QII: $\boxed{X(T) = 2X_1(T) + 3iX_2(T)}$

Q3.

$$f(t) = f(0)\, e^{-i\omega t}$$

$$(*) : \ddot{f}(t) + 3\dot{f}(t) - 4f(t) = 0$$

$$\dot{f}(t) = -i\omega f(0)\, e^{-i\omega t}$$
$$= -i\omega f(t)$$

$$\ddot{f}(t) = +(i\omega)^2 f(0)\, e^{-i\omega t}$$
$$= (i\omega)^2 f(t).$$

d'où $(*)$ devient : $f(t)\left[(i\omega)^2 - 3i\omega - 4\right] = 0.$

$$\Rightarrow \quad \omega^2 + 3i\omega + 4 = 0.$$

$$\Delta = (3i)^2 - 4\times 4$$
$$= -9 - 16$$
$$= -25$$
$$= (i5)^2$$

$$\omega_1 = \frac{-3i - i5}{2} \qquad \omega_2 = \frac{-3i + i5}{2}$$

$$\omega_1 = \frac{-8i}{2} \qquad \boxed{\omega_2 = i}$$

$$\boxed{\omega_1 = -4i}$$

QIV: f_1, f_2 deux solutions, $\beta, \gamma \in \mathbb{C}$

1 a) Oui,

$$(\beta \ddot{f}_1 + \gamma \ddot{f}_2) + \omega^2(\beta f_1 + \gamma f_2)$$

$$= \beta(\underbrace{\ddot{f}_1 + \omega^2 f_1}_{=0}) + \gamma(\underbrace{\ddot{f}_2 + \omega^2 f_2}_{=0})$$

$$= 0$$

1 b) Non, $F(T) = \beta F_1$

$$(\beta F_1)(\beta \ddot{F}_1) + \alpha(\beta \dot{F}_1)$$

$$= \beta^2 (\underbrace{F_1 \ddot{F}_1}_{-\alpha \dot{F}_1}) + \beta(\alpha \dot{F}_1)$$

$$= (\beta - \beta^2)\alpha \dot{F}_1 \neq 0$$

1 c) Oui,

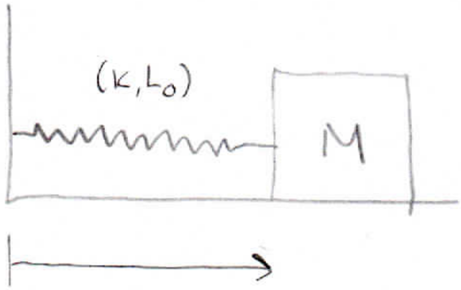
$$\ddot{(\beta F_1 + \gamma F_2)} + \dot{(\beta F_1 + \gamma F_2)} + X(T)(\beta F_1 + \gamma F_2)$$

$$= \beta(\ddot{F}_1 + \dot{F}_1 + X(T) F_1)$$

$$+ \gamma(\ddot{F}_2 + \dot{F}_2 + X(T) F_2)$$

$$= 0$$

Question longue /12



$\rightarrow \hat{U}_x$

2 a) $\vec{F}_{Ressort} = -K(X - L_0)\hat{U}_x$

$\vec{F}_{Friction} = -\mu_c M \dot{X} \hat{U}_x$

2 b) $\vec{F}_{TOT} = M\vec{a} = M\ddot{X}\hat{U}_x$

$\Rightarrow M\ddot{X}\hat{U}_x = -(K(x - L_0) + \mu_c M \dot{X})\hat{U}_x$

$\Rightarrow \ddot{X} + \mu_c \dot{X} + \frac{K}{M} X = \frac{K L_0}{M}$

$\Rightarrow \ddot{X} + \Gamma \dot{X} + \omega_0^2 X = y$

avec $\Gamma = \mu_c$, $\omega_0 = \sqrt{\frac{K}{M}}$, $y = \frac{K L_0}{M}$

2 c) On pose $X(T) = X_0 e^{-i\lambda T}$

$$\Rightarrow \dot{X} = -i\lambda X \quad , \quad \ddot{X} = -\lambda^2 X$$

Eq. (3) $\Rightarrow (-\lambda^2 - i\lambda\Gamma + \omega_0^2) X = 0$

$$\Rightarrow \lambda_\pm = \frac{i\Gamma \pm \sqrt{4\omega_0^2 - \Gamma^2}}{-2}$$

Critique $\Rightarrow \boxed{4\omega_0^2 = \Gamma^2}$

2 d) Sol générale = (sol. gén. prob. homogène i.e. $y=0$)

+ (sol. particulière)

Sol. particulière : On prends $X(T) = y/\omega_0^2$

$$\Rightarrow \dot{X}(T) = \ddot{X}(T) = 0$$

$$\Rightarrow \ddot{X} + \Gamma\dot{X} + \omega_0^2 X = 0 + 0 + \frac{\omega_0^2 y}{\omega_0^2} = y \quad \checkmark$$

$$\Rightarrow \boxed{\begin{gathered} X(T) = A e^{-i\lambda_+ T} + B e^{-i\lambda_- T} + y/\omega_0^2 \\ \lambda_\pm = -\frac{i\Gamma}{2} \mp \sqrt{\underbrace{\omega_0^2 - \Gamma^2/4}_{\geq 0}} \; ; \; A, B \in \mathbb{C} \end{gathered}}$$

2 e) $X(0) = L_0/2$, $\dot{X}(0) = 0$

$$\Rightarrow -i\lambda_+ A - i\lambda_- B = 0$$

$$\Rightarrow \boxed{A = -\frac{\lambda_-}{\lambda_+} B}$$

$$X(0) = A + B + g/\omega_0^2$$

$$= \underbrace{\left(\frac{\lambda_+ - \lambda_-}{\lambda_+}\right)} B + \underbrace{g/\omega_0^2}_{= L_0} = L_0/2$$

$$\Rightarrow \quad \frac{2\sqrt{\omega_0^2 - \Gamma^2/4}}{i\Gamma/2 + \sqrt{\omega_0^2 - \Gamma^2/4}}$$

$$\Rightarrow \boxed{B = -\frac{L_0}{4} \frac{\left(i\mu_c/2 + \sqrt{K/M - \mu_c^2/4}\right)}{\sqrt{K/M - \mu_c^2/4}}}$$

Pour les étudiants n'ayant pas réussi b), on peut donner des points partiels.