Ex.1

(0.5)

1) $\sin(2x) = \frac{1}{2} = \sin(\frac{\pi}{6}) \Leftrightarrow \begin{cases} 2x = \frac{\pi}{6} + 2k\pi \\ \text{ou} \\ 2x = \frac{5\pi}{6} + 2k\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{\pi}{12} + k\pi \\ x = \frac{5\pi}{12} + k\pi \end{cases}$ (1)

$\Rightarrow S = \{ \frac{\pi}{12} + k\pi, \frac{5\pi}{12} + k\pi \mid k \in \mathbb{Z} \}$

• Sur $[0, 2\pi]$: $\frac{\pi}{12}, \frac{13\pi}{12}, \frac{5\pi}{12}, \frac{17\pi}{12}$ (0.5)

(0.5)

2) $X = \cos x$: $2X^2 - 3X + 1 = 0$

$\Delta = 9 - 4(2)(1) = 1 \Rightarrow X_1 = \frac{3-1}{4} = \frac{1}{2}$ & $X_2 = \frac{3+1}{4} = 1$

↳ $\cos x = \frac{1}{2} = \cos \frac{\pi}{3} \Leftrightarrow x = \pm \frac{\pi}{3} + 2k\pi$

↳ $\cos x = 1 \Leftrightarrow x = 2k\pi$ (1)

$\Rightarrow S = \{ \pm \frac{\pi}{3} + 2k\pi, 2k\pi \mid k \in \mathbb{Z} \}$

• Sur $[0, 2\pi]$: $0, 2\pi, \frac{\pi}{3}, -\frac{\pi}{3} + 2\pi = \frac{5\pi}{3}$ (0.5)

Ex.2

↳ $a = 2{,}0\overline{63} \Rightarrow \begin{cases} 10a = 20{,}\overline{63} \\ 1000a = 2063.\overline{63} \end{cases} \Rightarrow 990a = 2043 \Rightarrow a = \frac{2043}{990}$ (2)

↳ $b = 3.\overline{54} \Rightarrow 100b = 354.\overline{54} \Rightarrow 99b = 351 \Rightarrow b = \frac{351}{99}$ (2)

Ex.3 $S \subset]-2, +\infty[$ (0.5)

↳ si $x - 1 < 0$: ✓ $\Rightarrow S_1 =]-2, 1[$ (0.5)

↳ si $x \geq 1$: $\sqrt{x+2} \geq x - 1 \geq 0 \Leftrightarrow x + 2 \geq x^2 - 2x + 1$ (1)

$\Leftrightarrow x^2 - 3x - 1 \leq 0$

$\Delta = 9 - 4(1)(-1) = 13$

$x_1 = \frac{3 - \sqrt{13}}{2}$, $x_2 = \frac{3 + \sqrt{13}}{2}$

x		x_1		x_2	
$x^2 - 3x - 1$	+	0	−	0	+

$S_2 = [1, +\infty[\cap [x_1, x_2] = [1, x_2]$

ccl : $S =]-2, x_2]$.

Ex. 4

1) $2\sum_{k=0}^{n}\binom{n}{k}k + \sum_{k=0}^{n}\binom{n}{k} = n2^n + 2^n = (n+1)2^n$

2

- $\sum_{k=0}^{n}\binom{n}{k}k = \sum_{k=1}^{n}\frac{n!}{(n-k)!\times k!}\times k = n\sum_{k=1}^{n}\frac{(n-1)!}{(n-k)!\,(k-1)!}$

$$= n\sum_{k=1}^{n}\binom{n-1}{k-1} = n\sum_{l=0}^{n-1}\binom{n-1}{l} = n2^{n-1}$$

- $\sum_{k=0}^{n}\binom{n}{k} = 2^n$

1

2) $\prod_{k=1}^{n}\left(1+\frac{k}{k+2}\right) = \prod_{k=1}^{n}\frac{2k+2}{k+2} = \frac{2^n\prod_{k=1}^{n}(k+1)}{\prod_{k=1}^{n}(k+2)} = \frac{2^n\prod_{l=2}^{n+1} l}{\prod_{l=3}^{n+2} l}$

$$= \frac{2^n\times 2}{n+2} = \frac{2^{n+1}}{n+2}$$

1

Ex. 5

↳ $\frac{x^2+3x+8}{x+1} - 6 = \frac{x^2+3x+8-6x-6}{x+1} = \frac{x^2-3x+2}{x+1} > 0$

$\Delta = 9-4(1)(2) = 1 \Rightarrow x_1 = \frac{3-1}{2} = 1$ & $x_2 = \frac{3+1}{2} = 2$

x		-1		1		2	
x^2-3x+2	$+$		$+$	0	$-$	0	$+$
$x+1$	$-$	0	$+$		$+$		$+$
$\frac{x^2-3x+2}{x+1}$	$-$	$\|\|$	$+$	0	$-$	0	$+$

$S_1 =]-1,1[\cup]2,+\infty[$

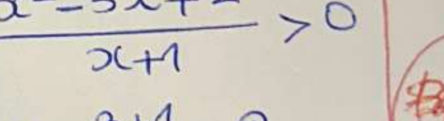
0.5

↳ $|x+1| \le 4 \iff -4 \le x+1 \le 4 \iff -5 \le x \le 3$

$S_2 = [-5,3]$

0.5

$\Rightarrow A = S_1 \cap S_2 =]-1,1[\cup]2,3]$



$\Rightarrow$ A est majoré par $x \in [3, +\infty[$
A est minoré par $x \in [-\infty, -1]$
$Sup(A) = 3 = max(A)$
$inf(A) = -1$
A n'admet de pas un min, car $inf(A) \notin A$

1

1

$\hookrightarrow \forall m, n \in \mathbb{N}^*$, $0 < \frac{m}{n+m} \Rightarrow 0$ est un minorant de $B \Rightarrow 0 \leq inf(B)$

$inf(B) \leq \frac{m}{n+m} \Rightarrow \lim_{n \to +\infty} inf(B) \leq \lim_{n \to +\infty} \frac{m}{n+m} = 0$

$\Rightarrow \boxed{inf(B) = 0}$

1

$\hookrightarrow \forall m, n \in \mathbb{N}^*$, $m < n+m \Rightarrow \frac{m}{n+m} < 1 \Rightarrow 1$ est un majorant de $B \Rightarrow Sup(B) \leq 1$

$\frac{m}{n+m} \leq Sup(B) \Rightarrow \lim_{m \to +\infty} \frac{m}{n+m} \leq \lim_{m \to +\infty} Sup(B) \Rightarrow 1 \leq Sup(B)$

$\Rightarrow \boxed{Sup(B) = 1}$

1

Ccl: B est majoré par $x \in [1, +\infty[$
B est minoré par $x \in]-\infty, 0]$

1

$inf(B) = 0$
$Sup(B) = 1$
$\forall m, n \in \mathbb{N}^*$, $\frac{m}{n+m} \neq 0 \Rightarrow \underset{inf(B)}{0} \notin B \Rightarrow B$ n'admet de pas un min.
$\forall m, n \in \mathbb{N}^*$, $\frac{m}{n+m} \neq 1 \Rightarrow \underset{Sup(B)}{1} \notin B \Rightarrow B$ ——— max.

1